# R & D NOTES

## Turbulent Diffusivities for Turbulent Pipe Flow

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Several recent papers have presented experimental data for the transport of heat with well-developed turbulent flow in a pipe. Brinkworth and Smith (1973) obtained data for water at a Reynolds number of 100,000 and concluded that the turbulent diffusivities for heat and momentum were equal. Sleicher, Awad, and Notter (1973) report data for the NaK eutectic mixture in the Reynolds number range of 26,000 to 302,000 with the conclusion that the ratio of the diffusivity of heat to momentum is less than 0.8 and is a function of Reynolds number. Chen (1973) also reports data for water but with a Reynolds number of about 270,000 and concludes that the diffusivity ratio of heat to momentum is 0.885. These comparisons are based upon different representations of the velocity profile to obtain the diffusivity of momentum. This note compares the data using the same velocity profile.

Brinkworth and Smith use a velocity profile model presented in their prior paper (1969):

$$\frac{du^+}{da} = -\left(\frac{1}{0.378}\right) 6a/[(1-a^2)(1+2a^2)] \quad (1)$$

This is applicable for the Reynolds number range of 50,000 to 350,000 and for the region $y^+ > 30$. This can be combined with Equation (2) to calculate a temperature profile.

$$\frac{(T-T_c)\rho C_p UR}{2q} = -\int_0^r \frac{1}{r\,\bar{\epsilon}_H}\left(\int_0^r urdr\right) dr \quad (2)$$

A computer program was prepared for this numerical integration and the temperature profile from the centerline to $y/R = 0.05$ was calculated. Equal eddy diffusivities for heat and momentum were assumed. A comparison between the calculated and experimental temperature profiles can then be made. Figure 1 shows the ratio of the calculated and experimental temperature differences between the radial locations and the centerline. Data of Sleicher, Awad, and Notter for Reynolds numbers of 79,000 and 302,000 the Chen data, and data reported by Buhr, Carr, and Balzhiser (1968) for the NaK eutectic at a Reynolds number of 97,200 and mercury at a Reynolds number of 124,000 are shown by the figure. Sleicher, Awad, and Notter question the high Reynolds number data of Buhr, Carr, and Balzhiser because of swirl flow contribution at these conditions. The two runs shown here are at flow conditions where this effect would not be a major contribution. Calculated data are observed to agree within a maximum of about 20% with experimental data. These results are consistent with the observation of Brinkworth and Smith and indicate equal diffusivities for heat and momentum for turbulent pipe flow.

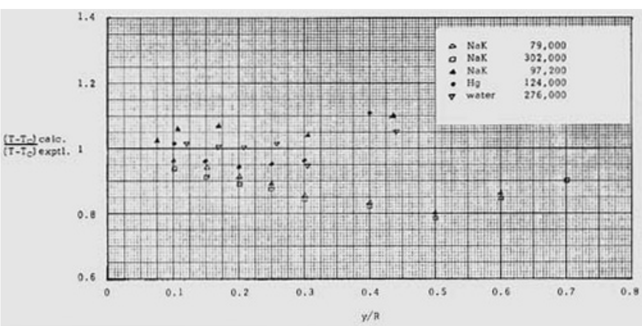


**Fig. 1. Temperature profile comparison.**

### NOTATION

$a$ = $y/R$
$C_p$ = specific heat
$q$ = wall heat flux
$r$ = radial distance from pipe centerline
$R$ = pipe radius
$T$ = temperature at any radial position
$T_c$ = centerline temperature
$U$ = average velocity
$u$ = axial velocity at any radial position
$u^*$ = shear velocity
$u^+$ = $u/u^*$
$y$ = radial distance from pipe wall
$y^+$ = $yu^*/\nu$

**Greek Letters**

$\bar{\epsilon}_H$ = total diffusivity for heat
$\nu$ = kinematic viscosity
$\rho$ = density

### LITERATURE CITED

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